SEMI-ANNUAL REPORT

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BOUNDS FOR THE ZEROS OF POLYNOMIALS

Principal Investigator: Richard J. Painter

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Department of Mathematics and Statistics

COLORADO STATE UNIVERSITY

Fort Collins, Colorado

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The notation, definitions, and theorem numbering used in this report are described in the project proposal and are not repeated here for the sake of brevity.

Theorem 3 (Ostrowski) gives conditions on n, α, and S which are necessary and sufficient for the existence of radii ρ and ρ^* such that for each polynomial $f(z)$ in class $C(n, \alpha, S)$ the "Rouché inequality" (1) is satisfied on $|z + a_1| = \rho$ and $|z| = \rho^*$. The three conditions A, B, and C are mutually exclusive but they are not exhaustive (for example, let n = 3, α = 5, S = 9). Moreover the "Rouché inequality" (1) is sufficient but certainly not necessary for $f(z)$ and $z^n + a_i z^{n-1}$ to have the same number of zeros interior to a simple closed curve. Hence it is appropriate to ask if there are values of n, α, S not satisfying A, B, or C but for which radii ρ and ρ^* do exist with the property that if $f(z)$ belongs to the class $C(n, \alpha, S)$ then the disc $|z + a_1| < \rho$ contains exactly one zero of $f(z)$ and $|z| < \rho^*$ contains exactly n-1 zeros of $f(z)$.

The principal investigator has been able to show that the answer to this question is "no", which is to say that Ostrowski's cases A, B, and C give the complete answer to the problem in terms of the parameters n, α, S. The proof is fairly straightforward, contains only one gimmick and contains no useful ideas. It is surprising that Professor Ostrowski overlooked this point, but it is not surprising that his results are unimprovable. This result falls under the category described in Sec. III a of the proposal.

A search of the literature since 1960 is in progress and has lead to the study of several recent results and one not so recent result on the bounds for eigenvalues of matrices. In particular the recent work of Richard Varga and an older paper of Alfred Brauer (1953) are of current interest and will lead, it is expected to a sharpening of the principal investigator's earlier

results along the lines described in Sec. III b of the proposal.

No new direction for the research is contemplated, but any new results in the proposed area may lead directly to analytic criteria for the irreducibility of monic polynomials with integer coefficients. Such sidelights will probably be of more academic than practical interest and hence no significant amount of the principal investigator's time will likely be devoted to this area.